

# Gauge factor of thick film resistors: outcomes of the variable range hopping model

C. Grimaldi, P. Ryser, S. Strässler

*Département de Microtechnique, IPM, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland.*

Despite a large amount of data and numerous theoretical proposals, the microscopic mechanism of transport in thick film resistors remains unclear. However, recent low temperature measurements point toward a possible variable range hopping mechanism of transport. Here we examine how such a mechanism affects the gauge factor of thick film resistors. We find that at sufficiently low temperatures  $T$ , for which the resistivity follows the Mott's law  $R(T) \sim \exp(T_0/T)^{1/4}$ , the gauge factor GF is proportional to  $(T_0/T)^{1/4}$ . Moreover, the inclusion of Coulomb gap effects leads to  $GF \sim (T_0'/T)^{1/2}$  at lower temperatures. In addition, we study a simple model which generalizes the variable range hopping mechanism by taking into account the finite mean inter-grain spacing. Our results suggest a possible experimental verification of the validity of the variable range hopping in thick film resistors.

## I. INTRODUCTION

Thick film resistors (TFRs) are composite materials in which metallic grains ( $\text{RuO}_2$ ,  $\text{Bi}_2\text{Ru}_2\text{O}_7$ , etc.) are embedded in an insulating glassy matrix. The characteristic transport properties of these materials render the TFRs particularly suitable as cryogenic thermometers and piezoresistive sensors. Despite this successful practical aspect, on the theoretical side the situation is less satisfying. The microscopic mechanism of transport in TFRs is in fact far from to be understood and several theoretical models have been proposed, none of them however being able to provide a completely satisfactory and definite answer.

Among the proposed transport mechanisms in TFRs, the most representative ones are those based on tunneling within a network of interconnected metallic clusters separated by thin glassy layers [1], hopping between isolated metallic grains [2] and phonon-assisted variable-range-hopping mechanism [3]. The first model should be consistent with a highly dishomogenous structure in which the metallic phase is organized mainly in large segments separated by thin layers of glass while the last two models are more suitable for homogeneous dispersions of separated metallic grains. The actual situation seems to lay somewhere in between these two structures. Recent extended x-ray absorption fine structure experiments in fact have revealed a bimodal distribution of the metallic particle sizes in  $\text{RuO}_2$ -based TFRs [4]. According to these measurements, the metallic phase is organized in large  $\text{RuO}_2$  grains or clusters with linear size ranging from  $\sim 200$  Å to  $\sim 6000$  Å and much smaller  $\text{RuO}_2$  grains with sizes of order 20-70 Å. Where the actual current path takes place is still under debate. However, It is plausible that transport in TFRs takes place mainly along paths connecting the small metallic grains, the large  $\text{RuO}_2$  clusters

being in fact too far away (a distance of order 100 Å as inferred from transmission electron microscopy [5]) from each other for an electron to directly hop from a large cluster to another large cluster.

TFRs have also a characteristic temperature dependence of the resistance  $R(T)$ . At around room temperature,  $R(T)$  has a minimum and it slightly increases at higher  $T$ . At lower temperatures,  $R(T)$  increases signalling a non-metallic behavior. In this low-temperature region,  $R(T)$  has been observed to follow a  $\exp(T_0/T)^x$  behavior with  $x \simeq 1/4$  [6] or  $x \simeq 1/2$  [7] depending on the room temperature resistance value of the samples. Recent measurements [8] have shown that  $R(T) \sim \exp(T_0/T)^x$  with  $x = 1/4$  with a cross-over to  $x = 1/2$  as the temperature is lowered for the most resistive samples. Such kind of temperature dependence is typical of Mott variable-range-hopping (VRH) mechanism of transport [3] affected by Coulomb interaction between grains (Coulomb gap effect [9]). Hence, the transport properties of TFRs at low temperatures seem to be qualitatively described by ordinary VRH as other disordered systems like  $n$ -type  $\text{CdSe}$  [10]. However, the values of  $T_0$  and  $T_0'$  that best fit the experimental data would lead to values of the optimal hopping length  $R_h$  of the same order of the localization length  $\xi$ , whereas the VRH mechanism should be observed only for  $R_h \gg \xi$  [8]. The validity of the VRH mechanism is therefore not clear and additional informations are required to analyse the validity of the VRH approach in describing transport in TFRs.

Here we propose that a quantity which can be helpful and easily measurable is the strain sensitivity, or gauge factor, and its temperature dependence. The gauge factor (GF) relates the variation of the total resistance  $R(T)$  with an applied strain  $\varepsilon$  and it is defined as follows:

$$GF = \frac{\delta R(T)}{\varepsilon R(T)}, \quad (1)$$

where  $\delta R(T)$  is the variation of  $R(T)$  under the applied strain  $\varepsilon$ . TFRs have high values of GF, typically between

2 and 35 at room temperature [11], and it is precisely this property that renders these systems particularly suitable for piezoresistive sensor applications. Here we are interested in studying how the VRH mechanism affects GF and its temperature dependence. We find that *if the low temperature transport is due to the VRH mechanism* then the gauge factor acquires a characteristic temperature dependence which could be experimentally determined.

In the next section we use the asymptotic regimes predicted by the VRH model to extract qualitative behaviors for  $GF$ . In Sec. III, a simple model is introduced with the aim of studying the effect of finite inter-grain distances and to test the validity of the qualitative behaviors of  $GF$ .

## II. QUALITATIVE RESULTS

In this section we analyze the transport properties of TFRs from the point of view of VRH model. To this end, we must assume drastic simplifications and we model therefore the TFRs structure as given by an homogeneous dispersion of small metallic grains (of typical sizes 20-70 Å) in the amorphous matrix and we neglect any possible effect of the larger metallic clusters on the bulk resistivity. As we discuss below, the assumption of homogeneity probably leads to a strong under-estimation of the room temperature GF value. However, instead of absolute values, here we are mainly interested in relative changes of GF as the temperature is varied.

An additional simplification we assume is to disregard the high temperature rise of  $R(T)$  beyond the temperature of minimum. Although the origin of this feature is still debated, it should nevertheless been given by electron-phonon intra-grain scattering, thermal expansion effects or maybe a combination of both. Within these approximations, at sufficiently high temperatures transport is governed by tunneling between adjacent grains which are separated by a mean distance  $d$ . In this high temperature regime therefore

$$R(T) \equiv R_0 \propto \exp(2d/\xi) \quad (2)$$

where  $\xi$  is the localization length and it is related to the barrier potential  $V$  between adjacent grains as  $\xi = \hbar/\sqrt{2mV}$ , where  $m$  is the electron mass. An applied strain modifies  $d$  and the resulting longitudinal GF becomes

$$GF \equiv GF_0 = 2d/\xi. \quad (3)$$

In the above expression it is implicitly assumed that the electrons hop along paths mostly parallel to the direction in which strain is applied. However, the actual microscopic paths are the result of hopping processes also along directions perpendicular to the applied strain. In this way, the longitudinal gauge factor is something less than

$2d/\xi$  and the amount of reduction depends on specific material properties. However, as shown in appendix A, such a reduction can be estimated by introducing a phenomenological parameter  $\chi$  which measures the percentage of hops along directions perpendicular to the strain, leading to the following expression for the longitudinal gauge factor:

$$GF_0 = \frac{1 - \chi/(1 - \nu)}{1 + \chi} 2d/\xi, \quad (4)$$

where  $\nu$  is the Poisson ratio which varies between 0.2 and 0.4 for typical TFRs [11]. As shown in appendix A,  $0 < \chi < 1/2$  and consequently  $0.1(2d/\xi) < GF_0 < 2d/\xi$ . Since  $\xi$  is of order of  $d$ , these values of  $GF_0$  are much less than those measured in TFRs ( $2 < GF_0 < 35$ ). We think that a strong enhancement of  $GF_0$  could be achieved when the assumption of homogeneity is relaxed. In fact, a strongly non-homogeneous distribution of metal grains in the glassy matrix could lead to a local strain distribution very different from the averaged one. In particular, the paths along which the current flows can enter regions of concentrated strain leading therefore to an enhanced total  $GF_0$ . Studies along this direction are currently under development.

Having obtained the high temperature resistance and strain sensitivity, we analyze now the situation at low temperatures. According to the VRH model, at sufficiently low temperatures hopping to adjacent grains is no longer favourable and  $R(T)$  acquires a temperature dependence. The question is whether the VRH mechanism of transport affects the low temperature regime of GF as well and if this variation can be experimentally tested.

In the VRH model, the transport properties are governed by the probability  $P_{ij}$  that an electron hops from grain  $i$  to grain  $j$  [3]:

$$P_{ij} \propto \exp \left[ -2r_{ij}/\xi - \frac{E(r_{ij})}{K_{BT}} \right], \quad (5)$$

where  $r_{ij}$  is the distance between grains  $i$  and  $j$  and  $E(r_{ij})$  is the energy threshold the electron experiences in hopping the distance  $r_{ij}$ . Assuming that a finite fraction of the grains is charged, the energy  $E(r_{ij})$  is made of two contributions [10]:

$$E(r_{ij}) = \frac{\beta}{gr_{ij}^3} + \frac{\gamma e^2}{\kappa r_{ij}}, \quad (6)$$

where  $g$  is the electron density of states,  $e$  is the electron charge,  $\kappa$  is the dielectric constant,  $\beta$  and  $\gamma$  are dimensionless constants. In the second right hand of equation (6), the first term describes the energy needed to hop to a grain a distance  $r_{ij}$ . This term is proportional to  $r_{ij}^{-3}$  because the probability an electron at site  $i$  has to find a site  $j$  with energy much nearer to its own scales as  $r_{ij}^3$ .

The second term in Eq. (6) describes the Coulomb energy due to finite charging of the grains, as it is expected by the fluctuations in the energy levels in the grains. [12] This term is responsible for the opening of the Coulomb gap. [9]

At sufficiently low temperatures, the resistance is governed by  $1/P_{\max}$  where  $P_{\max}$  is the maximum of  $P_{ij}$ . Hence, the optimization of the exponential in the hopping probability (5) leads to a temperature dependence of  $R(T)$  characterized by two cross-over temperatures:

$$T_0 = \frac{2048\beta}{27g\xi^3 K_B}, \quad (7)$$

which stems from the first term in the second hand side of Eq.(6), and

$$T'_0 = \frac{8\gamma e^2}{\kappa\xi K_B}, \quad (8)$$

which is given by the Coulomb interaction. Usually  $T'_0 < T_0$  and  $R(T) \sim \exp(T_0/T)^{1/4}$  for  $T'_0 < T < T_0$  while  $R(T) \sim \exp(T'_0/T)^{1/2}$  for  $T < T'_0$ .

Depending on the particular temperature range, the strain sensitivity of  $R(T)$  is governed by the strain sensitivity of  $T_0$  or  $T'_0$ . Let us first focus on the temperature range in which  $R(T) \sim \exp(T_0/T)^{1/4}$ . An applied strain  $\varepsilon$  leads to following relation between the resistance variation  $\delta R$  and  $\delta T_0$ :

$$\frac{\delta R}{R} = \frac{1}{4} \left( \frac{T_0}{T} \right)^{1/4} \frac{\delta T_0}{T_0}. \quad (9)$$

$T_0$  is inversely proportional to the electron density of states  $g$  which is proportional to the density of grains  $n$ . Assuming that an applied strain affects the density of metallic grains leaving the volume of the grains unchanged [13], we obtain  $\delta T_0/T_0 = -\delta g/g = \varepsilon(1-2\nu)$  and the gauge factor  $\text{GF} = \delta R/\varepsilon R$  becomes:

$$\text{GF} = \frac{1-2\nu}{4} \left( \frac{T_0}{T} \right)^{1/4}. \quad (10)$$

GF therefore increases as  $T^{-1/4}$  as the temperature is lowered provided  $T$  is in the range for which  $R(T) \sim \exp(T_0/T)^{1/4}$  holds true.

By further lowering the temperature, transport becomes affected by the Coulomb interaction and  $R(T)$  crosses towards the  $\sim \exp(T'_0/T)^{1/2}$  regime. Hence, under applied strain,  $\delta R/R$  reduces to:

$$\frac{\delta R}{R} = \frac{1}{2} \left( \frac{T'_0}{T} \right)^{1/2} \frac{\delta T'_0}{T'_0}. \quad (11)$$

The variation  $\delta T'_0$  is driven by the strain dependence of the total dielectric constant  $\kappa$  and, in full generality, it can be expressed as:

$$\frac{\delta T'_0}{T'_0} = -\frac{\delta \kappa}{\kappa} = \lambda \varepsilon (1-2\nu), \quad (12)$$

where  $\lambda$  is a dimensionless parameter and its value depends on the dielectric constants of the metallic and glassy phases. In appendix B we provide an explicit expression of  $\lambda$  based on the Maxwell-Garnett formula for the dielectric response of small metallic particles suspended in a dielectric. By using (11) and (12), the gauge factor in the Coulomb regime reduces to:

$$\text{GF} = \frac{1-2\nu}{2} \lambda \left( \frac{T'_0}{T} \right)^{1/2}. \quad (13)$$

The Coulomb effect is therefore reflected in a cross-over from a  $T^{-1/4}$  to a  $T^{-1/2}$  dependence of GF as the temperature is sufficiently lowered.

### III. SIMPLE MODEL FOR THE GF TEMPERATURE DEPENDENCE

Summarizing the main results obtained in the previous section, the temperature dependence of the resistance and the intrinsic GF as  $T$  is lowered is characterized, within the VRH theory, by three well distinguishable trends:

$$\begin{aligned} R &\sim \exp(2d/\xi) &\rightarrow \text{GF} &\sim 2d/\xi \\ R &\sim \exp\left(\frac{T_0}{T}\right)^{1/4} &\rightarrow \text{GF} &\sim \left(\frac{T_0}{T}\right)^{1/4} \\ R &\sim \exp\left(\frac{T'_0}{T}\right)^{1/2} &\rightarrow \text{GF} &\sim \left(\frac{T'_0}{T}\right)^{1/2} \end{aligned} \quad (14)$$

Note that for each behavior listed above, GF is proportional to  $\ln \rho(T)$ . The last two behaviors originate from the optimization of the inter-grain hopping probability  $P_{ij}$  of Eq.(5). It is possible in fact to define a universal functional form for  $R(T)$  which contains both  $\exp(T_0/T)^{1/4}$  and  $\exp(T'_0/T)^{1/2}$  as limiting values and which fits well transport data of Indium doped CdSe samples. [10] However, for sufficiently high temperatures, the optimization of  $P_{ij}$  requires optimal hopping distances of order or less than the mean distance  $d$  between adjacent grains. It would therefore be more correct to formulate the transport problem in such a way that the value of  $d$  is explicitly included. Here we provide a simple version of such a formulation capable of describing automatically the limiting behaviors listed in Eq.(14).

A simple general equation for the conductance  $G$  can be defined as follows. A single hopping process between grains at distance  $r_{ij}$  and with a hopping probability  $P_{ij}$  can be regarded as a resistive element with resistance  $R_{ij}$  proportional to  $1/P_{ij}$ . Hence, the total resistance can be constructed by interpreting paths characterized by different hopping distances as resistors in a parallel geometry. The resulting conductance should therefore be given by a weighted summation of the hopping probabilities  $P_{ij}$

for all values of  $r_{ij} \geq d$ . In this way, by employing a continuous approximation, we define the conductance  $G(T)$  as:

$$G(T) = a \int dr f(r) \exp \left[ -2r/\xi - \frac{E(r)}{K_B T} \right], \quad (15)$$

where  $E(r)$  is given in Eq.(6) and  $a$  is a constant introduced for dimensional purposes. In the above equation,  $f(r)$  is a weight function which takes into account the different path geometries. For our purposes, the detailed structure of  $f(r)$  is unnecessary and we retain only its main feature which is given by a lower cut-off representing the mean distance  $d$  between adjacent grains. Hence, we approximate  $f(r)$  by the step function  $\theta(r - d)$ . In this way, equation (15) reduces to:

$$G(T) = a \int_d^{+\infty} dr \exp \left( -2r/\xi - \frac{E(r)}{K_B T} \right). \quad (16)$$

For temperatures larger than

$$T_s = \frac{\xi E(d)}{2dK_B}, \quad (17)$$

equation (16) gives the proper high temperature limit (2), while for  $T \ll T_s$  the integral can be estimated by optimizing the exponential, leading therefore to the VRH regime. Concerning the gauge factor resulting from (16), it should be noted that an applied strain  $\varepsilon$  affects both the interval of integration and the energy threshold function  $E(r)$  through the electron density of states  $g$  and the dielectric constant  $\kappa$ . Therefore, by using the strain derivatives of  $g$  and  $\kappa$  introduced in the previous section, GF can be expressed as the sum of three different contributions,  $GF = GF_d + GF_g + GF_\kappa$  where:

$$GF_d = \frac{2d}{\xi} \left[ \frac{1 - \chi/(1 - \nu)}{1 + \chi} \right] \frac{G(T)}{G_0} e^{-E(d)/K_B T} \quad (18)$$

$$GF_g = \frac{1 - 2\nu}{K_B T} \frac{a}{G(T)} \int_d^{+\infty} dr \frac{\beta}{gr^3} \exp \left( -2r/\xi - \frac{E(r)}{K_B T} \right) \quad (19)$$

$$GF_\kappa = \frac{1 - 2\nu}{K_B T} \frac{\lambda a}{G(T)} \int_d^{+\infty} dr \frac{\gamma e^2}{\kappa r} \exp \left( -2r/\xi - \frac{E(r)}{K_B T} \right) \quad (20)$$

In equation (18)  $G_0 = a\xi \exp(-2d/\xi)/2$  is the asymptotic conductance in the high temperatures limit.

In Fig. 1a we plot the resistance  $R(T)$  obtained numerically from Eq.(16) in the absence of Coulomb repulsion ( $\gamma = 0$ ) for  $2d/\xi = 0.2, 2$ , and  $5$ . The resistance data are in units of the high temperature limit  $R_0 = 1/G_0$  and the temperature is given in units of

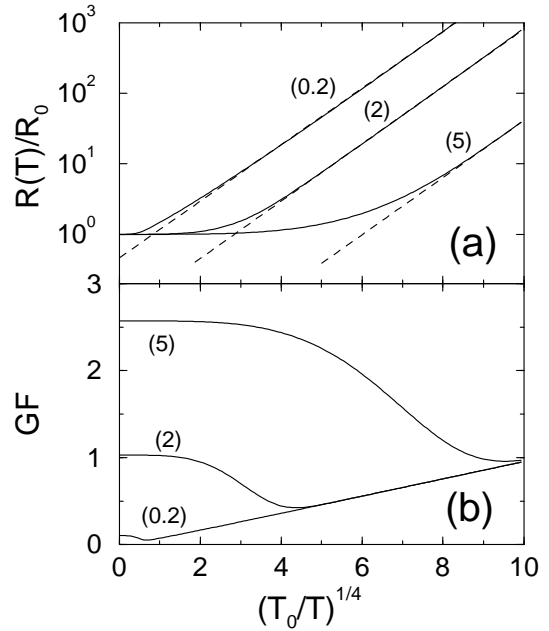


FIG. 1. (a): temperature dependence of the resistance calculated from Eq. (16) without Coulomb interaction ( $\gamma = 0$ ) and for the different values of  $2d/\xi$  reported in the parentheses (solid lines). The dashed lines fit the numerical data with  $\exp(\tilde{T}_0/T)^{1/4}$  where  $\tilde{T}_0 \simeq 0.72T_0$ . (b): corresponding temperature dependence of the gauge factor for the same values of  $2d/\xi$  as in Fig. 1a and for  $\nu = 0.3$  and  $\chi = 1/4$ .

$T_0$  [see Eq.(7)]. The curves show two distinct regimes as a function of temperature due to the finite values of the averaged inter-grain distance  $d$ . By using Eq.(17), the cross-over between these two regimes takes place roughly at  $T \sim T_s = 0.105T_0/(2d/\xi)^4$ , so that as  $2d/\xi$  increases the cross-over temperature  $T_s$  is reduced. For  $T > T_s$  [ $(T_0/T)^{1/4} < 1.755(2d/\xi)$ ] the resistance shows an activated regime while at sufficiently low temperatures [ $(T_0/T)^{1/4} > 1.755(2d/\xi)$ ] it becomes of VRH type. The latter behavior is signalled by the dashed lines which fit the resistance with  $R(T)/R_0 \propto \exp(\tilde{T}_0/T)^{1/4}$ , where  $\tilde{T}_0 \simeq 0.75T_0$ . The gauge factors GF associated to the resistance curves of Fig. 1a are plotted in Fig. 1b for  $\nu = 0.3$  and  $\chi = 1/4$ . A  $2d/\xi$  dependent cross-over is obtained for GF as well. For temperatures higher than  $T_s$ , GF is independent of  $T$  and approaches the limit  $GF_0$  given in Eq.(4). On the contrary, by lowering  $T$ , GF acquires a temperature dependence and for  $T \ll T_s$  increases as  $1/T^{1/4}$  in agreement with Eq.(10). From the results of Fig. 1b, the  $1/T^{1/4}$  dependence of the gauge factor holds true in a wide range of temperatures only for quite small values of  $GF_0$ .

The effect of Coulomb repulsion on  $R(T)$  and GF is displayed in Fig. 2 where the case  $2d/\xi = 2$  is investigated for different values of the Coulomb parameter  $T'_0$  defined in Eq.(8). In Fig. 2a, for  $T'_0/T_0 \neq 0$  the resistance approach the low temperature asymptotic regime

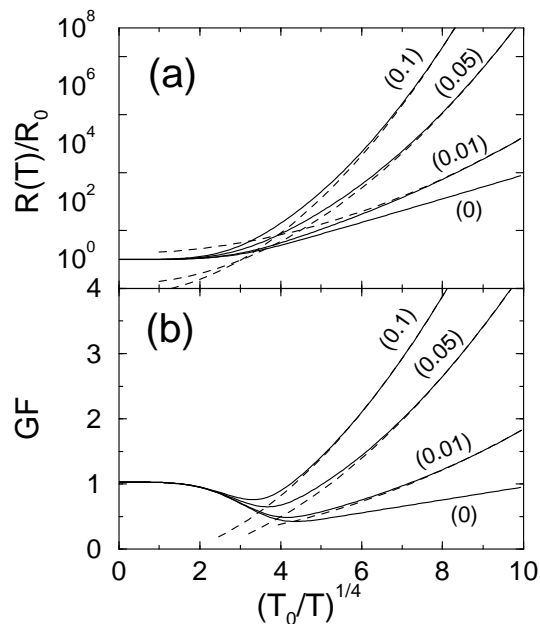


FIG. 2. (a): temperature dependence of the resistance with the inclusion of Coulomb interaction for  $2d/\xi = 2$  and  $T'_0/T_0$  as reported in the parentheses (solid lines). The dashed lines fit the numerical data with  $\exp(\tilde{T}'_0/T)^{1/2}$  where  $\tilde{T}'_0 \simeq 0.9T'_0$ . (b): temperature dependence of the gauge factor for the same values of  $T'_0/T_0$  as in Fig. 1a,  $\nu = 0.3$ ,  $\chi = 1/4$ , and  $\lambda = 5$  (solid lines). The dashed lines fit the low temperature regime of GF with  $(\tilde{T}'_0/T)^{1/2}$ .

$R(T)/R_0 \propto \exp(\tilde{T}'_0/T)^{1/2}$  with  $\tilde{T}'_0 \simeq 0.9T'_0$  (dashed lines). The Coulomb effect is reflected also in the temperature dependence of GF, Fig. 2b, where the curves have been calculated by using  $\lambda = 1$ ,  $\nu = 0.3$  and  $\chi = 1/4$ . The set of curves with  $T'_0 \neq 0$  show the low temperature asymptotic behavior  $GF \propto (\tilde{T}'_0/T)^{1/2}$  with values of  $\tilde{T}'_0$  somewhat larger than those obtained from the resistivity data ( $\tilde{T}'_0 \simeq 1.05T'_0$ ). Finally the temperature dependence of GF for different values of  $\lambda$  is plotted in Fig. 3 for the case  $2d/\xi = 2$  and  $T'_0/T_0 = 0.05$ .

#### IV. DISCUSSION AND CONCLUSIONS

The numerical solutions of equations (16)-(20) confirm the qualitative results given in the previous section, *i.e.*, as the resistance crosses over the VRH regime, the gauge factor acquires a temperature dependence which is qualitatively summarized in Eq.(14).

The model here presented is very simplified and susceptible of various improvements. For example, from statistical arguments, it would be certainly more correct to evaluate the temperature dependence of  $K(T)$  within the percolation theory for transport [14]. This could be achieved by explicitly taking into account in the evaluation of the critical path the lower cut-off  $d$  for the hopping distance and the strain dependence of  $g$  and  $\kappa$ .

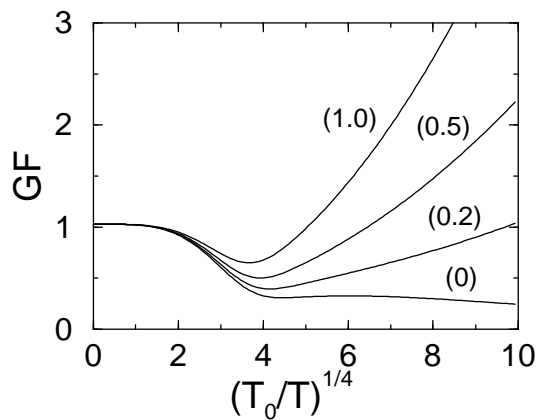


FIG. 3. Gauge factor calculated for  $T'_0/T_0 = 0.05$ ,  $\nu = 0.3$ ,  $\chi = 1/4$  and for different values of  $\lambda$  (reported in the parentheses).

As already stressed before, an additional simplifying assumption we have used is that of homogeneity. However the structure of TFRs is very complex, being made by a glass with dispersed metallic grains with bi-modal distribution of sizes. In this situation, under an applied strain, the stress distribution inside the sample could strongly modify GF as compared to the homogeneous case and to settle quantitatively this modification is certainly a very important issue.

A final open question regards the general applicability of the VRH concept when electrons hop between grains of finite size. In fact, in reality, the discrete electron energy levels of the grains are smeared by finite life-time effects given by the electron-phonon and electron-electron interactions together with the inter-grain tunneling coupling. At sufficiently low temperatures, the life-time effects are small and the discreteness of the energy levels requires phonon assisted hopping for electron transport. However, at sufficiently high temperatures, the energy smearing due to the finite electron life-time can be of the order of the level spacing. In this situation it can be easily shown that electrons can directly tunnel between grains without the assistance of phonons. Of course, a crucial role is played by the grain size which governs the energy level spacing. To our knowledge, the problem of how the VRH between grains is affected by the inclusion of finite life-time effects has never been addressed and its investigation is certainly of the most valuable interest.

In conclusion, the results here reported provide a clear prediction of the temperature dependence of the gauge factor in the hypothesis that transport is well described by the classical VRH theory. Hence, the simultaneous detection of the temperature dependences of the sheet resistance and the gauge factor can provide an useful experimental tool to test the validity of the VRH mechanism in TFRs. We note that some experimental data show that GF for commercial TFRs is largely independent of  $T$  from room down to cryogenics temperatures [19]. Our

analysis would indicate therefore that these experimental results are not compatible with the VRH mechanism of transport. However, it is important to stress that the VRH model predicts a strong temperature dependence of GF only when the resistance crosses over a  $\exp(T_0/T)^x$  type of regime ( $x = 1/2$  or  $1/4$ ), a test that has not been performed in Ref. [19]. Instead, a firm experimental test requires measurements of the temperature dependences of both  $R(T)$  and GF.

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## APPENDIX A:

In this appendix we estimate the reduction of the gauge factor due to the contributions of hopping processes perpendicular to the direction on which strain is applied. To this end, we first consider an operative definition of the gauge factor. Let us consider a cantilever beam in the  $x$ - $y$  plane with axis lying on the  $x$  direction on top of which is deposited the TFR. The thicknesses of the cantilever beam and of the TFR are measured in the  $z$  direction. According to elasticity theory, bending of the cantilever beam produces a finite strain  $\varepsilon_{xx} = \varepsilon$  in the  $x$  direction and no strain in  $y$  ( $\varepsilon_{yy} = 0$ ). Moreover, if  $\nu$  is the Poisson ratio, the strain of the TFR along the  $z$  direction is  $\varepsilon_{zz} = -\varepsilon\nu/(1-\nu)$ .

To measure the longitudinal GF of the TFR, a potential difference must be applied along the  $x$  direction. If the microscopic paths are made of discrete hopping processes exclusively along the direction of the field, the longitudinal GF would be equal to  $2d/\xi$  just as in Eq.(2). Conversely, the transversal GF would be zero because in this case the field is applied in the  $y$  direction for which  $\varepsilon_{yy} = 0$ . Let us consider now the situation in which there is a finite percentage  $\chi$  of hops along directions perpendicular to that of the applied field. The resulting resistance  $\tilde{R}_x$ , when the field is applied along the  $x$  direction, becomes  $\tilde{R}_x = (1-\chi)R_x + \chi(R_y + R_z)$ , where  $R_x$ ,  $R_y$  and  $R_z$  are resistances for hops in the  $x$ ,  $y$  and  $z$  directions, respectively. Although  $R_x$ ,  $R_y$  and  $R_z$  have the same average value, they differ when strain is applied. In fact:

$$\frac{\delta R_x}{\varepsilon_{xx} R_x} = \frac{2d}{\xi}; \quad \frac{\delta R_y}{\varepsilon_{xx} R_y} = 0; \quad \frac{\delta R_z}{\varepsilon_{xx} R_z} = -\frac{\nu}{1-\nu} \frac{2d}{\xi}. \quad (\text{A1})$$

Hence the longitudinal gauge factor  $\text{GF}_L$  becomes:

$$\begin{aligned} \text{GF}_L &\equiv \frac{\delta \tilde{R}_x}{\varepsilon_{xx} \tilde{R}_x} = \frac{(1-\chi)\delta R_x + \chi\delta R_z}{\varepsilon_{xx}[(1-\chi)R_x + \chi(R_y + R_z)]} \\ &\simeq \frac{1-\chi/(1-\nu)}{1+\chi} \frac{2d}{\xi}. \end{aligned} \quad (\text{A2})$$

By following the same reasoning, the transversal gauge factor  $\text{GF}_T$  can be estimated by realizing that the resistance for a field applied in the  $y$  direction is  $\tilde{R}_y = (1-\chi)R_y + \chi(R_x + R_z)$  and consequently:

$$\begin{aligned} \text{GF}_T &\equiv \frac{\delta \tilde{R}_y}{\varepsilon_{xx} \tilde{R}_y} = \frac{\chi(\delta R_x + \delta R_z)}{\varepsilon_{xx}[(1-\chi)R_y + \chi(R_x + R_z)]} \\ &\simeq \frac{\chi[1-\nu/(1-\nu)]}{1+\chi} \frac{2d}{\xi}. \end{aligned} \quad (\text{A3})$$

If the Poisson ratio  $\nu$  is known, the phenomenological parameter  $\chi$  can be estimated by measuring the ratio between the longitudinal and the transversal GFs which, according to (A2) and (A3), is given by:

$$\frac{\text{GF}_L}{\text{GF}_T} = \frac{1-\chi-\nu}{\chi(1-2\nu)}. \quad (\text{A4})$$

Experimentally this ratio is found to be larger than the unity and consequently the above expression leads to  $\chi < 1/2$ .

## APPENDIX B:

Here, we evaluate the strain dependence of the dielectric constant for TFRs. We model the structure as given by spherical metallic particles dispersed in a glassy matrix. The metallic and glassy phases occupy volumes  $V_m$  and  $V_g$ , respectively, so that the volume fraction of the metallic phase is  $\phi = V_m/(V_m + V_g) \simeq V_m/V_g$  for low metallic concentrations. Within the dipole approximation, the total dielectric constant  $\kappa$  is given by the Maxwell-Garnett formula [15,16]:

$$\kappa = \kappa_g \left[ 1 + \frac{3\phi}{1-\phi} \right], \quad (\text{B1})$$

where  $\kappa_g$  is the dielectric constant of the glass. For sufficiently small values of  $\phi$ , the variation  $\delta\kappa$  given by an applied strain  $\varepsilon$  is:

$$\delta\kappa = \kappa \frac{\delta\kappa_g}{\kappa_g} - 3\kappa_g \phi \varepsilon (1-2\nu), \quad (\text{B2})$$

where we have considered the metallic particles as perfectly rigid bodies compared to the glass so that  $\delta\phi = -\phi\varepsilon(1-2\nu)$  [13].

To find the variation  $\delta\kappa_g$  we should know an explicit expression for  $\kappa_g$  which is however a difficult problem. Nevertheless, experiments suggest that dielectric response of glasses is under several aspects quite similar to that of ionic solids and, in particular, it has been shown that the variation of the glass polarizability with pressure resembles that of the ionic crystalline compounds [16]. By starting with this latter observation we could argue that, for our purposes, the dielectric constant for simple glasses is roughly given by:

$$\kappa_g = \frac{1 + 8\pi\alpha/3\Omega}{1 - 4\pi\alpha/3\Omega}, \quad (\text{B3})$$

where  $\Omega$  has dimension of volume and  $\alpha \simeq 1/K$  is the ionic polarization with  $K$  being the elastic constant. An expression similar to (B3) is also recovered in theoretical analyses based on coarse-graining approach [17] (there,  $\Omega$  is a coarse-graining volume). An applied strain modifies  $\kappa_g$  via the volume  $\Omega$  and the elastic constant  $K$ . The strain dependence of the latter quantity can be guessed by observing that  $K = V''_{\text{ion}}(r_0)$  where  $V_{\text{ion}}(r)$  is the interaction energy of two ions a distance  $r$  apart and  $r_0$  is the equilibrium distance. A pedagogical model for  $V(r)$  is  $V(r) = -a/r + \exp(-br) + \text{const.}$  [18], so that one obtains  $K = V''_{\text{ion}}(r_0) \sim 1/r_0^2$ . This result suggests that  $\alpha/\Omega$  scales as  $1/[\text{distance}]$  and consequently from Eq.(B3) one obtains in a straightforward way:

$$\delta\kappa_g \simeq -\frac{(\kappa_g - 1)(\kappa_g + 2)}{3}\varepsilon(1 - 2\nu). \quad (\text{B4})$$

The above expression plugged into Eq.(B2) leads finally to:

$$\begin{aligned} \frac{\delta\kappa}{\kappa} &\simeq -\left[\frac{(\kappa_g - 1)(\kappa_g + 2)}{3\kappa_g} + 3\frac{\kappa_g}{\kappa}\phi\right]\varepsilon(1 - 2\nu) \\ &\equiv -\lambda\varepsilon(1 - 2\nu), \end{aligned} \quad (\text{B5})$$

where the last equality defines the dimensionless quantity  $\lambda$  used in the main text.

and it is expected that  $\text{RuO}_2$  has a value of  $E$  of the same order. Since typical glasses have  $E$  of order 70-100 GPa, it is a satisfying first approximation to consider the  $\text{RuO}_2$  grains as perfectly rigid compared to the embedding glass.

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